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LETTER TO THE EDITOR

Generation of electron entanglement in quantum dot systems

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Abstract

We propose a solid-state quantum structure capable of generating Einstein–Podolsky–Rosen (EPR) electron pairs using a parametric electron pumping idea and the Coulomb blockade phenomenon. The quantum structure consists of two coupled quantum dots and four leads. Our scheme is easy to implement, and it does not impose special requirements on the leads. By employing the parametric pumping idea, harmful processes can be avoided and only two quantum dots are needed. Furthermore, the EPR electron pairs are spatially separated.

(Some figures in this article are in colour only in the electronic version)

Quantum entanglement is a natural resource for quantum computation and quantum information [1]. The future development of quantum information and quantum computation depends largely on how effectively one can generate and process entangled states of different objects. So far, most experiments on quantum information processes, such as quantum cryptography [2] and quantum teleportation [3], have been done by using entangled EPR photon pairs. The entangled trapped ions and atoms also provide a means for the possible realization of quantum information and quantum computation [4, 5]. However, it should be more interesting and important in terms of applications to generate electron spin entanglement in solid state systems [6–14]. The usefulness of electron spin entanglement has been assured by the recent demonstration of long electron spin lifetime in semiconductors [15, 16]. Several methods [6–11, 17] for entanglement generation as well as entanglement detection in solid state systems have been proposed. Beam splitters [6], devices based on BCS superconductors [12, 13], and the two-dimensional interacting electron gas [14] are some examples. There are also proposals involving quantum dots (QDs) in generating entanglement

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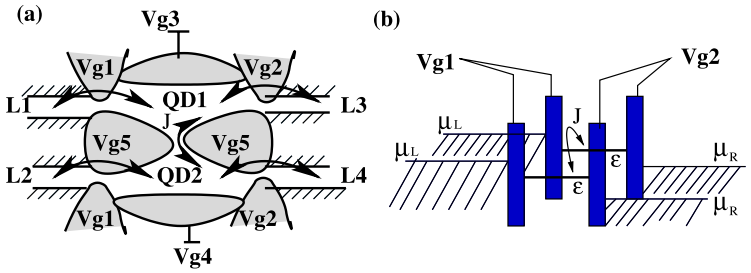

Figure 1. (a) Schematic diagram of a system with two coupled quantum dots (QD1 and QD2) and four leads. L1 and L2 are source leads. L3 and L4 are two drain leads. The gate voltages Vg1 and Vg2 control the potential barriers between the QDs and the leads. Vg3 and Vg4 are gate voltages which can tune the energy levels in the QDs while gate voltage Vg5 can adjust the coupling strength J between the two QDs. (b) Energy level diagram of our model. The energy levels the of QDs are between the chemical potentials of source and drain leads, $\mu_{\rm L} > \epsilon > \mu_{\rm R}$.

by taking advantage of the discreteness of the energy levels and large Coulomb interaction inside a QD [7–10]. For example, in [7], it was shown that only two electrons in the spin singlet state are allowed to pass through a QD because of the interference effects, but leads with narrow energy bands are required there. There are also proposals involving three QDs [8, 9] as well as two QDs [10].

In this letter, we propose a scheme of using two QDs to generate EPR pairs. The scheme is based on the Coulomb blockade phenomenon and a parametric pumping concept [18–21]. The ground state of two electrons located separately in two coupled quantum dots with strong intradot Coulomb interaction (Coulomb blockade) and negligible interdot Coulomb interaction is a spin singlet. Applying a periodically varying potential barriers between QDs and leads, two electrons are first injected into the QDs, where they relax to the spin singlet state; then they flow out of the QDs into the two drain leads spatially separated. Through a non-equilibrium Green function (NEGF) calculation, we find that the efficiency of entanglement generation is high with our scheme. The employment of the parametric pumping idea with very high barriers can avoid the forward and reverse transport processes that are harmful to the entanglement generation and processing. In contrast to the usual parametric pumping, the pumping process in our proposal does not need to work in the adiabatic regime. In contrast to other proposals [7, 8], our scheme imposes fewer requirements on the properties of the leads.

Our model consists of two QDs and four leads, as shown in figure 1. Each QD is connected to two leads, and J is the coupling coefficient between the two QDs. In figure 1(a), L1 and L2 are the electron source leads and L3 and L4 are the drain leads. The chemical potentials are $\mu_{\rm L}$ and $\mu_{\rm R}$ ($\mu_{\rm L} > \mu_{\rm R}$) for the source and drain leads, respectively (figure 1(b)). The dots are designed in such a way that there is only one relevant energy level in each dot with a double spin-degeneracy. The gate voltages Vg1 and Vg2 play the role of variable parameters in parametric pumping. They can be tuned to increase or decrease the barriers between the QDs and the leads, and thereby open or close the left and the right gates, so that electrons can flow into (out of) the dots. Gate voltages Vg3 and Vg4 are used to adjust the energy levels in QD1 and QD2, respectively. In our scheme, the values of Vg3 and Vg4 will be set in such a way that the two relevant energy levels have the same energy ϵ, and $\mu_{\rm L} > \epsilon > \mu_{\rm R}$. The coupling J is controlled by Vg5. The system is working in the Coulomb blockade regime; hence the on-site Coulomb interaction U is large in comparison with the level spacing of the QDs and the chemical potential difference between the leads; therefore the electrons prefer to stay in different dots.

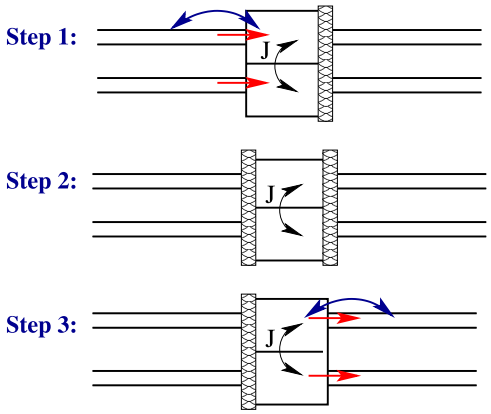

Figure 2. The schematic illustration of the operations. Step 1 is for the electron injection: Vg2 and Vg1 are tuned in such a way that the connection of dots with the left leads is open while that with the right leads is closed (shadowed strip). Electrons will flow from the source leads to the QDs. Step 2 is for the entanglement generation: both left and right barriers are high and electrons are trapped in the QDs. Nature will do the work and the system will relax to the spin-singlet ground state. So an EPR pair is generated. Step 3 is for extraction: open the right gate by lowering Vg2. The entangled electrons will flow into the two right drain leads separately.

To generate entangled electron pairs, we will vary two parameters Vg1 and Vg2 in three steps, as illustrated in figure 2. In the first step, the left barrier is lowered such that electrons can flow into the QDs from the source leads while the right barrier is increased to shut down the connection between the QDs and the right leads, or 'close' the right gate (denoted by the shadowed strip in figure 2). Since the system is in the Coulomb blockade regime, more than one electrons in each QD is prohibited. Altogether there are two electrons in the two QDs. In the second step we will close the left gate by raising Vg1 while Vg2 is kept unchanged (right gate remains closed). The electrons in the two QDs can be treated as an isolated system since they are completely trapped in the QDs. The ground state of the two electron system is a spin singlet state. Through the spontaneous emission of phonons or photons the electrons will relax to the ground state, and a spin singlet is generated in the QDs. In the last step the electron pair will flow into the drain by lowering the right barrier to open the right gate. We will show that the two electrons will preferably tunnel to different leads to generate a spatially separated EPR pair because of the large Coulomb interaction U.

In the second step, the system consists of two electrons and two isolated coupled QD systems. Its proper Hamiltonian can be written as $H_1 = \sum_{l,\sigma} \epsilon a^\dagger_{l\sigma} a_{l\sigma} + \sum_\sigma (J a^\dagger_{1\sigma} a_{2\sigma} + \text{H.c.}) + \sum_l U n_{l\uparrow} n_{l\downarrow}$, where $a_{l\sigma}$ ($a^\dagger_{l\sigma}$) is the electron annihilation (creation) operator in QD l ($l = 1, 2$), and $n_{ls} = a^\dagger_{ls} a_{ls}$ ($s = \uparrow, \downarrow$). ϵ is the on-site energy. J is the coupling coefficient between QDs. U is the intradot Coulomb interaction while the interdot Coulomb interaction is negligible. In our model $U + (\epsilon - \mu_{\rm L}) >$ several Γs, where Γ describes the coupling of leads and QDs.

In contrast to two electrons in an atom where the lower energy level is normally a spin triplet due to the strong e–e interaction, the ground state of a double-quantum-dot system with one electron in each QD is a spin singlet, as demonstrated in [22, 23]. This is due to the negligible interdot but strong intradot Coulomb interaction U as well as Pauli's principle. An electron in the spin singlet can virtually tunnel to a QD occupied by the other electron when the exchange coupling J is nonzero. This virtual tunnelling is not allowed for a spin triplet state because of Pauli's principle; hence the spin singlet state has lower energy. The energy

difference depends on the value of J, which is tuned by Vg5 in our model. Raising J will cause a larger energy difference in spin singlet and triplet states.

In the third step, the right gate is open such that the electrons in the two QDs will flow into the drain leads L3 and L4. We shall assume that the gate is opened suddenly, and the electron wavefunction will not be affected by the gate opening (sudden approximation). In order to model the system at this stage, one needs to add some terms in the Hamiltonian to describe the tunnelling process and leads. If the leads are non-interacting and non-magnetic, the proper Hamiltonian is

$$\begin{aligned} H &= H_1 + \sum_{l,k,\sigma} \epsilon_{lk} b^\dagger_{lk\sigma} b_{lk\sigma} + H_{\mathrm{T}}, \\ H_{\mathrm{T}} &= \sum_{k,\sigma} (V_k a^\dagger_1 b_{\mathrm{L3},k,\sigma} + V_k a^\dagger_2 b_{\mathrm{L4},k,\sigma}) + \mathrm{H.c.}, \end{aligned} \tag{1}$$

where $b_{lk\sigma}$ and $b^\dagger_{lk\sigma}$ are the electron creation and annihilation operators in lead l. V_k describes the coupling between QD and the leads. There is no coupling between QD1 and lead L4 and between QD2 and L3. In our calculation, $H = H_0 + V$, where $V = H_{\mathrm{T}} + \sum_\sigma (J a^\dagger_{1\sigma} a_{2\sigma} + \mathrm{H.c.})$ is the perturbation, and H_0 is the unperturbed Hamiltonian. We shall use a transition matrix (T-matrix) formalism [7, 24] to calculate the probability for electrons to tunnel from the QDs to the same lead and to two different leads.

The transition rate from the initial state $|\phi_{\mathrm{i}}\rangle$ with energy ε_{i} to the final state $|\phi_{\mathrm{f}}\rangle$ is

$$w_{\mathrm{fi}} = \frac{2\pi}{\hbar} |\langle \phi_{\mathrm{f}} | T | \phi_{\mathrm{i}} \rangle|^2 \, \rho(E_{\mathrm{f}})|_{E_{\mathrm{f}} \approx \varepsilon_{\mathrm{i}}}, \tag{2}$$

where $T = V + \lim_{\eta \to 0} V \frac{1}{\varepsilon_{\mathrm{i}} - H_0 + \mathrm{i}\eta} T$ is the transition matrix operator, and $\rho(E_{\mathrm{f}}) = \rho_0$ is the density of states in the drain leads at E_{f}. The tunnelling of electrons from QDs to leads is dominated by the resonant tunnelling, and one has to calculate the transition rate to all orders in H_{T} [25]. Under the conditions of $U \gg J$ and $\Gamma \gg J$, we have

$$\frac{w_{\mathrm{fi}}}{w'_{\mathrm{fi}}} \sim 2\frac{\Gamma^2}{J^2} \gg 1, \tag{3}$$

where w_{fi} denotes the transition rate of electron pair tunnelling to different leads, while w'_{fi} is that to the same lead. Equation (3) implies that the unwanted process (w'_{fi}) is suppressed and the electron pair in the spin singlet state preferably tunnel to different leads; thus a spatially separated EPR electron pair is generated.

Beyond the above perturbation theory, the time-dependent current from QD1 to L3 can be evaluated more accurately from the non-equilibrium Green function (NEGF) theory [26]:

$$I(t) = 2\mathrm{e} \int_{-\infty}^{t} \mathrm{d}t_1 \int \frac{\mathrm{d}\omega}{2\pi} \mathrm{Im}\{\mathrm{e}^{-\mathrm{i}\omega(t_1 - t)} \Gamma(\omega) \theta(t) \theta(t_1) [G^<_{11}(t, t_1) + f(\omega) G^{\mathrm{r}}_{11}(t, t_1)]\}, \tag{4}$$

where $\Gamma(\omega) = 2\pi \sum_k |V_k|^2 \delta(\omega - \epsilon_k)$ and θ is the step function. f is the Fermi–Dirac distribution function of electrons in lead L3. $G^<$ (G^{r}) is the standard lesser (retarded) Green function [26], and the subscript l ($l = 1, 2$) is for QD l. In the following analysis, the Hartree–Fock mean field theory is used to treat the Coulomb interaction U, with $U n_{l\uparrow} n_{l\downarrow} \approx U \langle n_{l\uparrow} \rangle n_{l\downarrow} + U n_{l\uparrow} \langle n_{l\downarrow} \rangle$. Under this approximation, the Green functions $G^{\mathrm{r}}(t, t_1)$ and $G^<(t, t_1)$ can easy be obtained via the Dyson equation and the Keldysh equation: $G^{\mathrm{r}} = g^{\mathrm{r}} + g^{\mathrm{r}} \Sigma^{\mathrm{r}} G^{\mathrm{r}}$ and $G^< = G^{\mathrm{r}} \Sigma^< G^{\mathrm{a}} + (1 + G^{\mathrm{r}} \Sigma^{\mathrm{r}}) g^< (1 + \Sigma^{\mathrm{a}} G^{\mathrm{a}})$, where the self-energy $\Sigma^{\mathrm{r}}_{lj}(t, t_1) = -\frac{\mathrm{i}}{2} \Gamma \theta(t) \delta(t - t_1) \delta_{lj}$, $\Sigma^<_{lj}(t, t_1) = \sum_k |V_k|^2 i f(\epsilon_k) \mathrm{e}^{-\mathrm{i}\epsilon_k (t - t_1)} \delta_{lj}$, and $g^{r(<)}$ are the retarded (less) Green functions without coupling to leads L3 and L4 (i.e. $V_k = 0$).

The resulting current is shown in figure 3, in which the right gate (controlled by Vg2) is opened at $t = 0$. The different curves in the figure correspond to different values of J.

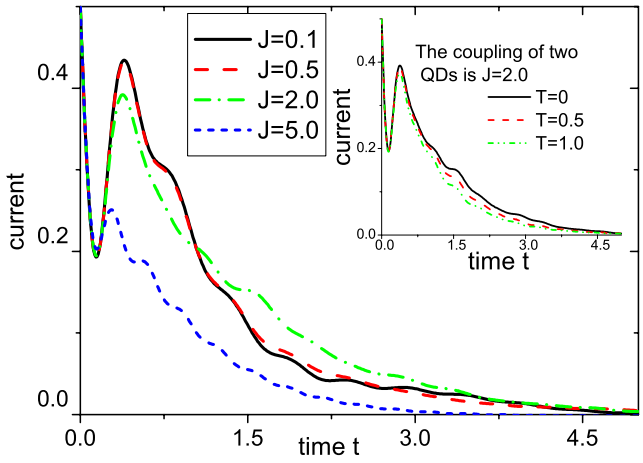

Figure 3. Current versus time t for different J with $T = 0$. Inset: current I versus t for $J = 2.0$ with various temperatures $T = 0$, 0.5 and 1. The other parameters are $U = 10, \epsilon = 2, \mu_R = 0$ and $\Gamma(\omega) = \Gamma\theta(\omega + 10)$ with $\Gamma = 1$. The initial number of spin-up (spin-down) electron in each QD is $n_0 = 0.5$. The time t is in the unit of $\hbar/\Gamma$, and the temperature T is in units of Γ/k_B.

The currents for a fixed $J = 2.0$ and various temperatures are shown in the inset of figure 3. They all exhibit oscillation behaviour, and the differences between these curves are small. In fact, the oscillation of the current originates from quantum interference of the two levels in the QDs in the tunnelling process. In particular, figure 3 clearly shows that the timescale for electrons to tunnel out from QDs is $\hbar/\Gamma$. The magnitude of Γ is typically of the order of 1–0.1 meV,[4] thus the tunnelling timescale of $\hbar/\Gamma$ is about 10^{-11}–10^{-12} s. The response time of electrons in solids is determined by the plasma frequency and RC relaxation time. In a typical semiconductor, the plasma oscillation time is 10^{-11} s or smaller [28], and a typical RC time is less than 10^{-10} s [21]. The speed of varying gate-voltages is also important. With recent advances in material science and nanotechnology, this timescale can be shorter than 10^{-11} s, as is indicated by the recent invention of the 604 GHz transistor [29]. In our scheme, Vg1 and Vg2 are periodically varied in order to continuously generate entangled current; one period in our model is the sum of three timescales (three steps). The timescale for two electrons flowing into QDs (step 1) is the same as that of flowing out (step 3), and they will be determined by the largest of $\hbar/\Gamma$, the inverse of the plasma frequency and the RC relaxation time (including the time required to apply a gate-voltage), i.e., less than 10^{-10} s. The timescale for step 2 will be the relaxation time of two electrons from the excited state (ES) (spin triplet) to their ground state (GS) (spin singlet). The relaxation time of electrons with spin-flip from ES to GS of QD(s) has been investigated in experiments [16, 30, 31]. Its experimental value covers a wide spectrum, ranging from milliseconds [30] to nanoseconds [16, 31]. What the upper and lower limits of the spin relaxation time are is not clear at present. However, one may learn something from atomic physics. It is known that the lifetime of some metastable atoms can easily be of the order of minutes (hydrogen at the 2s orbit for example), but this cannot rule out the fact of lifetimes of tens of nanoseconds for many other excited atoms. One way to eliminate the spin relaxation effect is to make electron pairs tunnel only to the spin singlet state. Theoretically, this can be done if the chemical potential μ_L of the source leads always lies between the excited and ground states, i.e. $E_{GS} < \mu_L < E_{ES}$.

[4] $\Gamma < U - \mu + \epsilon$, while the reported value of U is given in, for example, [27].

However, the energy levels of QDs are affected by the variation of the QD–lead coupling. Furthermore, the difference between the ES and GS energies is small; thus it is hard to realize this idea in practice. In our proposal, we need electrons capable of relaxing from the spin triplet excited states to the singlet ground state quickly enough. With currently available materials and technology, the overall timescale of one cycle in our scheme can be shorter than 10^{-8} s. Thus, the rate of EPR pair generation should not be a bottleneck for quantum information processes in comparison with the typical clock speed of GHz of modern classical computers.

Another feature of figure 3 is that the areas enclosed by the x-axis and curves for different coupling J, corresponding to the number of out-flow electrons, are not equal, implying non-zero remaining electrons in QDs even at infinite time. This is because of the expansion of the energy level of the order of Γ in QDs due to lead–QD coupling. The remaining electrons are harmful to the entangler, but the probability can be greatly reduced by the increase of $\epsilon - \mu$.

Our proposal uses a parametric pumping concept and the Coulomb blockade phenomena. The gate voltages Vg1 and Vg2 are two controlling parameters. Instead of calling it a turnstile, we use the term parametric pumping here because entangled electron current will be generated even at zero bias between source and drain leads. In the Coulomb blockade regime, only one electron is allowed in each QD so that the entangled electrons will flow into different leads. Although a larger U favours spatial separation of entangled electrons in step 3, it gives a smaller difference between spin triplet and singlet states such that it may take a longer time to create a spin singlet state in step 2. It should be pointed out that there is already experimental knowledge of how to use the Coulomb blockade to add electrons one by one to a quantum dot, starting from a zero number of electrons.

In contrast to an entangler using one QD [7], our scheme imposes fewer requirements on the properties of the leads. In contrast to an entangler using triple QDs [8], in our scheme, harmful forward and reverse tunnelling processes are reduced by the large potential barriers created by Vg1 and Vg2 such that the leakage current can be neglected [21]. Hence, our model is easier to implement experimentally. We would also like to discuss the differences between this work and that of Hu and Das Sarma [10]. Hu and Das Sarma [10] compiled a set of conditions for an entangler with two QDs from some heuristic arguments. Then, implicitly assuming that two entangled electrons in two QDs would be unentangled if they cannot tunnel out of the QDs at the same time ('synchronization'), they conclude, however without quantitative demonstration of its feasibility, that one can still use two QDs to generate entangled electron pairs by implementing the turnstile idea developed by others many years ago. In the current work, we carry out a detailed NEGF calculation to show that one can use two quantum dots to generate EPR electron pairs by varying the periodically potential barriers between the QDs and the leads, the parametric pumping idea studied in recent years. In [10], the importance of spin relaxation from a triplet state to the spin singlet state was not illustrated. This point is fully discussed now. We show that electrons will relax to their ground state (spin singlet) in QDs in step 2 without the influence of electrons in the leads. And the process of this step leads to a natural spin singlet ground state for the two electrons. The Keldysh–Green function formalism is very useful in handling non-equilibrium transport. It normally gives a much more accurate result in comparison with other methods. In particular, it can take into account the interaction effect of electrons in the QDs and leads. The obtained results should provide insight into entanglement generation and good guidance for experimentalists.

In summary, we have proposed an EPR electron pair entangler of two quantum dots based on a parametric pumping concept and the Coulomb blockade effect. Through calculations, we show that unentangled electrons from source leads to two coupled quantum dots become entangled EPR pairs. Then the entangled electrons are spatially separated by flowing out of

the quantum dots into different drain leads. Furthermore, we show that EPR pairs can be generated at a rate of GHz.

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